

A Personal Guide to Electronic Slot Machines

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PREFACE

Gambling is a fundamental human activity. People have always gambled, are gambling and will continue to gamble in the future. It is a feeling of adventure that seeks change, a search for the unknown and chance, and all that is knew. It sends us to the casinos, space research and exploration, the laboratory and financial activities, the stock market. It is a part of human nature. We enjoy the thrill of risk taking. We decide not to carry an umbrella, betting that it will not rain, and feel good when it doesn't. We exceed the speed limit, "taking a chance" that a patrolman will not catch us and issue a ticket. We over extend ourselves financially in the hope that a raise is on the horizon. People with a family history of cancer continue to smoke taking the chance that it will not affect their health. We all take chances and gamble to some extend. This "impulse" is part of us and the more we understand it the better we can enjoy our lives and deal with uncertainties.

The purpose of this book is to give the reader a basic knowledge and explanation of how modern slot machine operates and understands the odds of playing slot machines. Virtually, anyone who visits a casino is familiar with a slot machine and how it operates, even a novice. You insert a coin, pull the handle or press a button and wait a few seconds to see if you win. It's basic simplicity accounts for much of the success of slot machines in today's casinos.

First, it is unreasonable to claim that this book will make you a winner at slot machines, though I would hope that the insights provided will have a positive outlook in that direction. There exits no tricks or optimum strategy, which will make you a winner. However, a better understanding of how slot machines operates and how calculations of theoretical win probabilities are produce may affect your playing and hopefully increase your odds.

In Chapter One, a brief historical summary of the development of slot machines is introduced with several comments on gambling. Chapter Two

deals with the basic mathematics of permutations, combinations and probability. We show the basic concepts of how to calculate pay out percentages and generalize it to a popular modern slot machine. Chapter III goes to the heart of a slot machine. A detail description is given without revealing any proprietary secrets of the industry. Chapter IV discusses the psychological effects of playing with a slot machine and suggestions for maintaining control and good sense while gambling with slot machine.

This book does not judge the morality of gambling though I am a enthusiastic player, I am even more enthusiastic about the design and outcome of an electronic slot machine. It is the latter enthusiasm that I hope will be carried by the reader.

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CHAPTER 1

Gambling and the Slot Machine

A Short History

Gambling is a fundamental human activity. In the early history of man, prehistoric existence was a struggle of gambles against nature with the ultimate prize of survival. Today, with daily routines, man's necessity to gamble is getting involved in the action of every day living. It is a feeling of adventure that seeks changes, a search for the unknown and a fascination with games of chance. In random games such as bingo and slot machines, there is an equal chance for everyone to perhaps win for the first time in their life. Gambling has become a vital part of some lives and in some cases has lead to destructive and potentially addictive consequences while in others a delight and a diversion for modern man and woman.

Charles Fey, (1862-1944), is considered the Thomas Edison of Slots. He created his first machine in 1895, the same year X-rays were discovered by Wilham Roentgen, in the basement of his home, in San Francisco. He placed a few on location and did so well that he devoted full time to inventing, manufacturing and operating slot machines. In 1896, he opened a factory in San Francisco where he created many machines including the famous Liberty Bell, Draw Poker, Three Spindle and the Klondike. The business was destroyed in the 1906 Earthquake and Fire. Today, the original site is memorialized as a California Historical Landmark 937.

The first three reel slot machine created by Charles Fey in 1898 was called the Liberty Bell Slot. This machine is the standard of over one million machine worldwide. The Liberty Bell is owned by Marshall and Frank Fey, the sons of Charles Fey and can be seen at the Liberty Bell Saloon and Restaurant in Reno, Nevada.

Besides the nation's largest public display of vintage slot machines, many original items such as leaded glass shades, numerous beer tray, firearms, old photographs, Edison phonographs and massive cash registers can be seen. In addition to preserving the old slots produced by his father, Marshall Fey in 1983 wrote "Slot Machines: A Pictorial History of the First 100 years of the World's Most Popular Coin-Operated Gaming Device". The book is now in his fifth printing.

The term slot machine was originally used for automatic vending machines as well as for the gambling devices, but in the 20th century, the term became restricted to gambling devices. The first such gambling device in the US were mere novelties that did not return coins but presented opportunities, such as two toy horses that would race after a coin was inserted. By the end of the nineteenth century, machines that paid off in coins were in existence, usually in the form of a circular display with a spinning indicator that came to rest or pointed to a number, color, or picture. Today's most popular slot machines are 3 reel forms with a window showing winning combinations for coins played into the machine.

In the 1920's, the forces of morality and their law opposed the operation of slot machines. It was believed that the distribution of slot machines were controlled by organized crime and legislation restricted their sale. Nevada was the only state where legalized gambling was allowed. As other states and countries permitted gambling, the slot machines came into wide use throughout the world. US manufacturers such as International Gaming Technology had a prime segment of the markets. For the slot enthusiast, the action of pulling the handle, the sound of the reels falling into line and the jingle of cascading coins were all part of the attraction. By the late 1970's, electronic machines operated by push button and having visual displays were common. In the 1980's, the popularity of slot machines and table games were running even, but by the 90's, slots had taken over and now account for over two thirds of casino revenue in the US. Slot machines gained their universal appeal in casino because unlike the other games, they are played at the pace of the player and don't require any specific skills. Commonly referred to as *one-armed bandit*, the goal of the game is to spin the reels so that symbols on all reels line up on a single line. If this happens, the slot machine pays out coins according to the pay off table posted on the front of the cabinet. The pay off table describes the amount based on how many coins have been played and the type of symbols lining up. Players have a much greater chance of hitting any of the lower paying combinations.

Types of Machines

There are two kinds of slot machines: straight slots and progressives. These machines are essentially the same, but for one feature: straight slots will pay the winner a predetermined amount, where as progressive slots have a jackpot that grows by a percentage of each coin played. In general, a number of progressive machines are linked to a single computer where a grouping of machines feed into one jackpot, and the jackpot grows to a very large dollar sum, hundred of thousands and even millions. These grouping make up a number of casinos with many machines throughout several states.

Progressive machines can be identified by flashing electronic pay off signs displayed either on top of the machines or above a group of machines.

There are many different types of machines to choose from. Whether playing a progressive or straight slot, players will be faced with a number of choices. Machines vary on denominations of coins, the number of reels, how many coins to play, and single or multiple pay lines. The wages can be as low as a nickel and up to \$ 500.00 per coin. The most common machines are three reels types but some have four and five reels. When it comes to the odds of winning, one must look at the number of reels a machine has, and the jackpot pay off. The higher the number of reels and the higher the jackpot, the probability of hitting the jackpot decreases considerably. A computer programs the combination of symbols that will line up on a single line. In Chapter three, a detail description of a typical three-reel machine will be presented. Most machines are set out to pay from 83 percent up to 98 percent of the coins played averaged over a long period of time.

Most popular machines operate on a two and three coins maximum play. With some machines, the number of coins played is proportional to the pay off and with others the pay off is more than proportional. For example, the IGT popular Double Diamond machine jackpot pays 800 coins for one coin play and 1600 coins for two coins play, while the Triple Diamond slot machine pays a jackpot of 1000 coins for a single play and 2500 coins for the maximum two coin play. Another machine, called the "Wheel of fortune", requires that you play the maximum coins for you to win the jackpot. We have mentioned that most slot machines have one pay line but

three, five and nine lines are not uncommon. For these machines, the maximum coins have to be played in order to optimize a likely win.

Most casinos will place their liberal machines where they are likely to attract the most attention such as heavy traffic areas and in locations visible from many directions on the casino floor. This may or may not be the case everywhere. In the next chapter, we present some basic concepts of permutations and combination and show how to calculate the probability of winning with a three-reel slot machine. An introduction to random numbers and their use in determining a jackpot will be discussed.

Chapter II

The Mathematics of Slot Machines

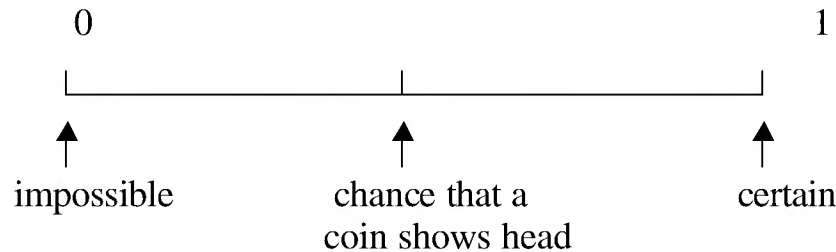
An electronic device called a random number generator determines today's electronic slot machines outcomes. Slot machines are programmed to reflect the desired pay out percentage determined by the casino, which cannot be set below the legal minimum. The Gambling Commission in the state to which the machine is delivered determines legal minimums. Some casinos advertise a 99% pay back on \$1.00 machines. What this means is that the casino will hold the difference between the stated pay back and 100% as their profit margin on all monies played on that machine. It does not mean that at any one time a player will get back 99% of the coins he put into the machine. So what is a random number generator and how does it work? Before we answer this question, it will be useful to review some basic concepts of probability and learn the fundamentals of permutations and combinations. These concepts will help us in understanding how to derive pay out percentages and establish hit and win frequencies for a given slot machine.

Probability

Probability is a branch of mathematics that deals with determining the likelihood that an event will have a particular outcome. It is based on the study of permutations and combinations and is the foundation for statistics. Two French mathematicians, Blaise Pascal and Pierre de Fermat are credited with the development of probability theory in the 17th-century, but one can go back as early as the 12th century B.C., where Chinese records indicate studies of square arrays of numbers with the property that the rows, columns, and diagonals add up to the same number. Over the centuries, the interest of probability theory grew in attempts to answer questions arising in games of chance such as how many times a pair of dice must be thrown before the chance that a six will appear is 50/50. A number between 0 and 1, inclusive, represents the probability of outcome with a probability 0 for an

event that will not occur and probability of 1 indicating certainty that it will occur.

If we represent impossibility by zero and certainty by one, we can represent probability as a line between zero and one. This is shown below.



Everything that is impossible is placed, at zero a person is not a bird, and everything that is certain is placed at one, people are mortals. The whole point of probability is to discuss uncertain events before they occur. After this event, things are completely different. When the weather forecast gives a 90% chance of rain, it may in fact not rain. The chance that an earthquake may happen this week in San Francisco is very small but it may occur today.

Before delving into the mathematics, we should ask what determines probability. Physics tells us that all events in nature are subject to physical laws, that their outcomes are therefore dictated by these laws and that, by correct application of these laws, we should be able to determine these outcomes. For instance, if a coin of a certain weight is flipped with a certain force, the laws of physics should tell us how many times it would turn over before it comes to rest on a certain surface and therefore, which side will be up. In reality, the problem may not contain all the information needed to solve it and chance comes into the solution. Part of the chance element in flipping a coin comes from a lack of information such as which side of the coin is up before the flip. In games of chance involving cards, coin, roulette wheels, etc. a random event is often called a “fair” event. In Slot machines random numbers provide a way for the game to be fair. The probability of selecting a winning combination is only based on the random number selected by the computer chip. There are very few aspects of life, the universe, or anything, in which chance is not in some way crucial to existence. It is becoming increasingly difficult to succeed in life whether one runs a business, practices medicine, deals in finances or engineering without a keen appreciation of chance and probability. For this reason if you are

going to play slot machines, you need to know what your risks and chances are and this is the goal of this chapter.

Permutations and Combinations

Many problems in probability require some methods of counting. For example, how many five letter words, meaningful and meaningless, can be written with the 26 characters of the alphabet? For the first letter of word we have a choice among 26 different character. For the second letter we have the same choice, that is, $26 \times 26 = 26^2$ possibilities. For the third letter we have again a choice among the 26 characters. Hence the total number of three letter words is $26^2 \times 26 = 26^3$. For a five letter word we have then 26^5 . In general for n sorts of objects when an unlimited number of objects of each sort is available, the total number of ways we can find k distinct spaces, each with one object is given by

$$n \times n \times \dots \times n = n^k$$

Now let's look at k distinct spaces and n different objects without repetition and assume that $n \geq k$. The first space can be filled in n different ways. For the second space, only $(n-1)$ objects are available and the first two spaces can be filled in

$$n (n - 1) \text{ ways}$$

For the third space, we have $(n - 2)$ remaining objects for each consecutive space the number of available objects decreases by 1. For the k th space, the choice is among

$$n - k + 1$$

objects. The total number of possibilities to fill the k spaces with n objects is therefore

$${}_n P_k = n (n - 1) (n - 2) \dots (n - k + 1) \quad (1)$$

If the number n objects is equal to the number of spaces, we obtain an important special case. Each arrangement is simply called a *permutation of the n objects*. From (1) it follows that they are

$${}_n P_n = n (n - 1) (n - 2) \dots (3)(2)(1)$$

$$= n ! \quad (2)$$

! is the mathematical symbol for factorial and $n !$ is read as "n factorial"

Note: By definition $0 ! = 1$

Example 1: How many possible orderings are there in the letters a,b,c?

$n=3$ and the permutation of 3 letters is calculated from equation (2)

$$3 ! = 3 \times 2 \times 1 = 6$$

The six orders are listed below

abc acb bac bca cab cba

Each of these arrangements is called a permutation of the letters, a, b, c. Suppose we want to find the number of ways to arrange the three letters a,b,c in different two-letter groups where ab is different from ba and there are no repeated letters?

Because order matters, we are finding the number of permutations of size 2 that can be taken from a set of size 3. We can list them as:

ab, ba, ca, ac, bc, cb

An arrangement of a set of n objects where order is important is called a permutation of the n objects. An arrangement of $k \leq n$ objects in a given order is called a permutation of n objects taken k at a time.

Multiplying equation (1) by $(n - k)(n - k - 1) \dots 3. 2. 1$ which is the same as $(n - k) !$, we have

$$\begin{aligned} {}_n P_k &= \{n(n - 1) \dots (n - k + 1)(n - k) \dots 2.1\} / (n - k) ! \\ &= n ! / (n - k) ! \end{aligned} \quad (2)$$

Example 2: Determine the number of permutations of seven objects taken five at a time .

$$\begin{aligned}
{}_7P_5 &= 7! / (7 - 5)! = (7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1) / (2 \times 1) \\
&= 7 \times 6 \times 5 \times 4 \times 3 \\
&= 2,520
\end{aligned}$$

Note: the slash shown here is the division sign. For example $9! / 5!$ is read 9 factorial divided by 5 factorial

We have seen in the previous discussion that permutations take into account the order of objects in a group. However, when the order is not important, we need to determine the number of combinations of n items taken k at a time. A combination of n objects taken k at a time is any subset of k times from the original n . In a study, participants are asked to select three letters from a list of four letters. How many possible grouping of three letters could a given participant select? To answer this, suppose we label the letters W, X, Y and Z.

Then we can list the combinations as follows:

Combinations	Permutations
W,X,Y	WXY, WYX, XWY, XYW, YXW, YWX
W,X,Z	WXZ, WZX, XWZ, XZW, ZXW, ZWX
W,Y,Z	WYZ, WZX, ZWY, ZYW, YWZ, YZW
X,Y,Z	XYZ, XZY, ZXY, ZYX, YXZ, YZX

We see that for each of the four possible combinations there are ${}_3P_3 = 3! = 6$ permutations. Using notations for combinations like that for permutations we get:

$$3! {}_4C_3 = {}_4P_3$$

To find the number of combinations without listing them, we use:

$$\begin{aligned}
{}_4C_3 &= {}_4P_3 / 3! \\
&= 24 / 6 \\
&= 4
\end{aligned}$$

In general notation we have the following formula for combinations:

$$\begin{aligned} {}_nC_k &= {}_nP_r / k! = \{ n! / (n - k)! \} / k! \\ &= n! / [k! (n - k)!] \end{aligned} \quad (3)$$

Example 3: How many ways can we select three letters from the letters of RSTUV?

$n=5$ $k=3$

$$\begin{aligned} {}_5C_3 &= 5! / (3! (5 - 3)!) \\ &= (5 \times 4 \times 3 \times 2 \times 1) / (3 \times 2 \times 1 (2 \times 1)) \\ &= 10 \end{aligned}$$

and they are: RST, RSU, RSV, RTU, RTV, RUV, STU, SUV, TUV, and TVU

Example 4: How many combinations of the letters a b c d e are there taken:

a) 5 at one time

$${}_5C_5 = 5! / ((5 - 5)! \times 5!) = 1$$

b) 2 at a time

$${}_5C_2 = 5! / ((5 - 2) \times 2!) = 5 \times 4 / 2 = 10$$

Example 5: In how many ways can 3 seven's come up in 8 spins of a roulette wheel?

A typical configuration for three 7's might be the 1st, 4th, and 6th spins of the wheel. Here we are interested in 8 sets of slots which can be fitted by 7's. Order is unimportant here, since, switching the seven's from the 1st to the 6th will not result in a different event. Hence, the number of combination is

$${}_8C_3 = \frac{8!}{3!(8-3)!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

Example 6: Find the probability of a full house (three of a kind and two of another kind), say three kings and two aces

Let A be the event "full house" then $P(A) = n_A / n$

Where A = the total probability of getting a "full house", n = the total number of outcomes (of selecting 5 cards out of 52)

$$n = 52! / (5!(52-5)!) = (52 \times 51 \times 50 \times 49 \times 48) / (5 \times 4 \times 3 \times 2 \times 1) \\ = 2,598,960$$

The number of ways to be given three aces is ${}_4C_3$, that is

$${}_4C_3 = 4! / (3!(4-3)!) \\ = (4 \times 3 \times 2 \times 1) / (3 \times 2 \times 1)(4-3) \\ = 4$$

and the number of ways to be given two kings is ${}_4C_2$ or

$${}_4C_2 = 4! / (2!(4-2)!) \\ = (4 \times 3 \times 2 \times 1) / (2 \times 1)(2 \times 1)$$

$$= 6$$

The combined number of ways to be given three aces and two kings is

$${}_4C_3 \times {}_4C_2 = 6 \times 4 = 24$$

$$P(a) = n A / n = k ({}_4C_3 \times {}_4C_2) / {}_{52}C_5$$

Where k is the number of different types of full houses (three of a kind and two of a kind),

With cards

2,3,4,5,6,7,8,9,10, J,Q,K,A

There are 13 x 12 possible types of full houses and k = 156

$$P(A) = 156 \times {}_4C_3 \times {}_4C_2 / {}_{52}C_5$$

$$= 156 \times 24 / 2,598,960$$

$$= 0.00144$$

Slot Machines and Combinations

Looking at the front of a slot machine cabinet, a pay table and description of winning combinations of symbols such as 7's, Triple Bars, Double Bars, single Bars, Blanks and Cherries are brightly displayed. This information is crucial if we want to determine how pay out percentages are generated for a given slot machine. There are many different playing machines and each one has its own game characteristics and pay out tables for a winning probabilities. In order to better understand the probabilities of getting a winning combination of symbols on a single line, we will apply some of the basic concepts of permutations and combinations we have discussed up to this point.

Let's say that on a three reel machine, the pay table for the symbol of cherries is as follow:

Symbols	pay out (# of coins)
Ch Ch Ch	45
Any two Cherries	18
Any one Cherry	6

In how many ways can cherry symbols combine with any other symbols on a three reel pay line machine?

There is only one way for all three cherries to appear on a single line.

Ch Ch Ch

The next possible arrangements of cherries is as follows: we have one item to be displayed on three windows, that is, the total number of permutation is given by $n!$

Here $n=3$ and using equation (1), we have $3! = 3 \times 2 \times 1 = 6$

The six permutations are listed below:

Ch Ch Any
Any Any Ch
Ch Any Ch
Any Ch Any
Ch Any Any
Any Any Ch

(Note: **Any** stands for any other symbols on the physical reel)

Hence, there are 6 ways that cherries align on the pay line

If we at look at bar symbols such as the TB (*triple bar*), DB (*double bar*), SB (*single bar*), the total number of combinations for all three types of symbols aligning on the pay line is listed below:

Single Bars	Double Bars	Triple Bars
R1 R2 R3	R1 R2 R3	R1 R2 R3
SB SB SB	DB DB DB	TB TB TB
SB SB DB	DB DB SB	TB TB SB
SB DB SB	DB SB DB	TB SB TB
SB SB TB	DB DB TB	TB TB DB
SB TB SB	DB TB DB	TB DB TB
SB DB TB	DB TB SB	TB SB DB
SB TB DB	DB SB TB	TB DB SB
SB DB DB	DB SB SB	TB DB DB
SB TB TB	DB TB TB	TB SB SB

There are 27 ways of aligning any bars on the pay line.

Some slot machines, have a "wild" symbol added to the reels. These symbols not only substitute for any other symbols on the line but can also double or triple the pay out of the combination it is substituting for. These "wild" symbols increase the number of ways symbols line up on a single line:

Let us derive all the possible combinations for a slot machine containing a "wild" symbols such as a 3X which triples the value of the win. First, let us look at a pay table that has a 3X symbol and their corresponding winning amounts :

SYMBOLS			PAY OUT
3X	3X	3X	2500
7	7	7	200
TB	TB	TB	80
DB	DB	DB	40
SB	SB	SB	20
Any bar	Any bar	Any bar	10
Two 3X's		blank	20
One 3X		two blanks	4

Table I. Pay Out Table for Wild 3X symbols with two coins

Let's start with similar symbols. As we said earlier, there will be only one way to align three like symbols on a single line, that is,

3X	3X	3X
7	7	7
TB	TB	TB
DB	DB	DB
SB	SB	SB
Blank	Blank	Blank

There are 3 ways of combining a 3X with a single bar

3X	SB	SB
SB	3X	SB
SB	SB	3X

And the same for two x 3X's wild symbols,

3X	3X	SB
3X	3X	SB
SB	3X	3X

There are then six ways for any symbols to combine with a single 3X or two 3X symbols.

Applying the same concepts to the other symbols (SB,DB,TB, 7's and blanks), we have $6 \times 5 = 30$ combinations. Now we are ready to sum all possible combinations:

A) Number of similar symbol combinations on a single line (6)

3X	3X	3X
7	7	7
TB	TB	TB
DB	DB	DB

SB SB SB
Blank Blank Blank

B) Number of combinations for any symbols with 3X's (48)

Since there are 6 ways of combining 3X's with two SB's and two 3X's with one SB:

3X SB SB
SB 3X SB
SB SB 3X

3X 3X SB
3X SB 3X
SB 3X 3X

With 5 different symbols (7,TB,DB,SB,and Blank), the total number of combinations is just

$$6 \times 5 = 30$$

In addition, any two bars can also combine with a 3X symbols as well, that is:

3X SB DB SB 3X DB SB DB 3X
3X DB SB DB 3X SB DB SB 3X

3X DB TB DB 3X TB DB TB 3X
3X TB DB TB 3X DB TB DB 3X

3X SB TB SB 3X TB SB TB 3X
3X TB SB TB 3X SB TB SB 3X

The total number of ways 3X's that can combine with any two bars is 18. The total number of combinations for any symbols with a 3 X is 48.

C) Number of combinations for any bars (24)

Single Bars	Double Bars	Triple Bars
SB SB DB	DB DB SB	TB TB SB
SB DB SB	DB SB DB	TB SB TB
SB SB TB	DB DB TB	TB TB DB
SB TB SB	DB TB DB	TB DB TB
SB DB TB	DB TB SB	TB SB DB
SB TB DB	DB SB TB	TB DB SB
SB DB DB	DB SB SB	TB DB DB
SB TB TB	DB TB TB	TB SB SB

In this example , we have shown that there are 78 ways of arranging any winning symbols listed on the pay out table. This information will be necessary in deriving calculations of pay out percentages. It is interesting to note that today's slot machines make it possible to use the many combinations of symbols possible by using random numbers to select the pay out probabilities and adding an unlimited number of "virtual" stops to map these winning combinations.

The next analysis will deal with the calculation of a percentage pay out based on Table I and the symbols we have created above. A physical reel strip generally consists of 22 positions or “stops”. The symbols on the reel are placed to correspond with certain Stop positions. Between each symbol is a space referred to as a “Blank”. Blanks are treated as symbols. Referring to our pay y table for the 3X slot machines, there will be 11 “Blank” with the rest of our symbols making up the remaining 11 Stops for a total of 22 Stops.

Our reel strips will have the following Stops:

Stop	Reel 1	Reel 2	Reel 3
1	Blank	Blank	Blank
2	3X	3X	3X
3	Blank	Blank	Blank
4	7	7	7
5	Blank	Blank	Blank
6	TB	TB	TB
7	Blank	Blank	Blank
8	DB	DB	DB
9	Blank	Blank	Blank
10	Blank	Blank	Blank
11	SB	SB	SB
12	Blank	Blank	Blank
13	SB	SB	SB
14	Blank	Blank	Blank
15	DB	DB	DB
16	Blank	Blank	Blank
17	TB	TB	TB
18	SB	SB	SB
19	Blank	Blank	Blank
20	SB	SB	SB
21	DB	DB	DB
22	Blank	Blank	Blank

Next we determine the number of possible hits for every combination in order to calculate the pay outs in one cycle. We do this by setting up a table that will describe the number of hits per reel and pay out per cycle of play. We are using the pay table containing the wild symbol 3X. Blanks do not return any coins.

Pay Table Calculation Sheet

WINNING COMBINATIONS			HITS PER REEL			HITS	PAYS	TOTAL
3X	3X	3X	1	1	1	1	2500	2500
7	7	7	1	1	1	1	200	200
3X	3X	7	1	1	1	1	1800	1800
7	3X	3X	1	1	1	1	1800	1800
3X	7	3X	1	1	1	1	1800	1800
3X	3X	TB	1	1	2	2	720	1440
TB	3X	3X	2	1	1	2	720	1440
3X	TB	3X	1	2	1	2	720	1440
3X	3X	DB	1	1	3	3	360	1080
DB	3X	3X	3	1	1	3	360	1080
3X	DB	3X	1	3	1	3	360	1080
3X	3X	SB	1	1	4	4	180	720
SB	3X	3X	4	1	1	4	180	720
3X	SB	3X	1	4	4	4	180	720
3X	7	7	1	1	1	1	600	600
7	3X	7	1	1	1	1	600	600
7	7	3X	1	1	1	1	600	600
3X	TB	TB	1	2	2	4	240	960
TB	3X	TB	1	1	1	1	240	960
TB	TB	3X	2	2	1	4	240	960
3X	DB	DB	1	3	3	9	120	1080
DB	3X	DB	3	1	3	9	120	1080
DB	DB	3X	3	3	1	9	120	1080
3X	SB	SB	1	4	4	16	60	960
SB	3X	SB	4	1	4	16	60	960
SB	SB	3X	4	4	1	16	60	960
3X	BLANK	3X	1	11	1	11	20	220
3X	3X	BLANK	1	1	11	11	20	220
BLANK	3X	3X	11	1	1	11	20	220
3X	BLANK	BLANK	1	11	11	121	10	1210
BLANK	3X	BLANK	11	1	11	121	10	1210
BLANK	BLANK	3X	11	11	1	121	10	1210
3X	ANY BAR	ANY BAR	9	9	9	729	30	21870
BLANK	BLANK	BLANK	11	11	11	1331	0	0
ANY BAR	3X	ANY BAR	9	9	9	729	30	21870
ANY BAR	ANY BAR	3X	9	9	9	729	30	21870
TB	TB	TB	2	2	2	8	40	320
DB	DB	DB	3	3	3	27	20	540
SB	SB	SB	4	4	4	64	10	640

Total 4,346 15,200 107,310

Pay Out Percentage:

With 22 stop positions on each reel, there will be a total of $22 \times 22 \times 22 = 10,648$ cycles generated by the mechanical slot machine. The pay out percentage is calculated as follow:

Cycle: 10,648

Pays = 107,310

$$\begin{aligned}\text{Pay Out Percentage} &= (\text{pays/cycle}) \times 100 \\ &= (107,310/10,648) \times 100 \\ &= 1008 \%\end{aligned}$$

$$\begin{aligned}\text{Hold Percentage} &= 100 - \text{Pay Out Percentage} \\ &= 100 - 1008 \\ &= -908 \%\end{aligned}$$

The hit frequency is calculated to be

Number of hits: 168

$$\begin{aligned}\text{Hit Frequency} &= (\text{Number of Hits/ Number of Cycle}) \times 100 \\ &= (168/10,648) \times 100\end{aligned}$$

$$= 40.8 \%$$

The Win Frequency is equal to one divided by the Hit Frequency

$$\text{Win Frequency} = 1/(\text{Hit Frequency})$$

$$= (1/ 40.8) \times 100$$

$$= 2.451$$

With this type of machine, the player will win and the casino will loose money. The pay out is greater than the number of cycles and the slot machine will generate no profit for the casino, unless the pay out is drastically reduced. In order to make money, the number of cycles will have to be increased. For this to happen and maintain the look and feel of a real mechanical slot machine, the industry came up with an electro-mechanical device, that is, a slot machine that controls the winning percentage for the casino.

This is achieved with a computer inside the slot machine which controls the actual slot game as well as the theoretical pay out percentage. High tech stepper motors, similar to those used in the space industry, operate the reels. A stepper motor is a special type of motor that is capable of stopping at a precise, predetermined point in its rotation. It is this ability that makes the modern slot machine possible. The reel symbols come to rest at their exact location as determined by the MPU control board. Mechanical devices inside the machine enable physical reels to spins and control the pay out of coins and reject invalid coins. A detailed discussion of the inside of slot machine will be introduced in the next chapter.

As we have seen, the mechanical slot machine with 22 physical stops has not enough cycles to generate a high pay out and is further limited by its physical size. Larger machines and increased number of reels have shown to be unpopular and perceived to have an unfair advantage. Microprocessor have changed the nature of slot machines and have created an unlimited amount of stops per reel, regardless of physical limitations.

Returning to our pay table as a reference, the only way to increase the hit frequency and lower the win frequency is to add more cycles to the reel.

This can be readily achieved by extending our physical reel to include "soft stops" or virtual stops. The microprocessor must randomly generate a number that will fall between two physical stops. This is easily done with random number generators. The standard stops per reel in the industry are 32, 64, 72, 128 and 256. The table below give the number of cycles generated by different number of virtual stops.

No. of Stops per Reel	Number of Cycles
(32x32x32)	32,768
(64x64x64)	262,144
(72x72x72)	373,248
(128x128x128)	2,097,152
(256x256x256)	16,777,216

Win Combinations for Virtual Reels

Thus far, we have shown how to calculate the percentage pay out for a mechanical slot machine having 22 stops per reel. In this section we will look at a physical reel that has 128 virtual stops. These multiple positions within the virtual reel are assigned to the same position upon the actual reel. A virtual reel table provides a mechanism for varying the odds of occurrence for an actual reel position by assigning different numbers of virtual reel positions to each actual reel position. For instance, the jackpot "3X" may be assigned to one virtual reel position, that is one random number. During the operation, the virtual reel obtains a number from the random number generator and uses this number as an index into the virtual reel table to obtain a corresponding actual reel position. The standard physical reel within the machine is stopped at the indexed actual reel position. This process is repeated for every reel within the machine and the outcome is then determined from the chosen reel positions. Again, referring to our original pay table (see Table I), we generate all possible win combinations for a 128 virtual reel stops.

Pay Table Calculation Sheet

R1	R2	R3	# of Hits Per Reel			TOTAL	PAY	TOTAL
						HITS		
3X	3X	3X	1	1	1	1	2500	2500
7	7	7	6	8	7	336	200	67200
3X	3X	7	1	1	7	7	1800	12600
7	3X	3X	6	1	1	6	1800	10800
3X	7	3X	1	8	1	8	1800	14400
3X	7	7	1	8	7	56	600	33600
7	3X	7	6	1	7	42	600	25200
7	7	3X	6	8	1	48	600	28800
3X	3X	TB	1	1	8	8	720	5760
TB	3X	3X	8	1	1	8	720	5760
3X	TB	3X	1	8	1	8	720	5760
3X	TB	TB	1	8	8	64	240	15360
TB	3X	TB	8	1	8	64	240	15360
TB	TB	3X	8	8	1	64	240	15360
3X	3X	DB	1	1	14	14	360	5040
DB	3X	3X	14	1	1	14	360	5040
DB	3X	3X	14	1	1	14	360	5040
3X	DB	DB	1	13	14	182	120	21840
DB	3X	DB	14	1	14	196	120	23520
DB	DB	3X	14	13	1	182	120	21840
3X	3X	SB	1	1	25	25	180	4500
SB	3X	3X	25	1	1	25	180	4500
3X	SB	3X	1	21	1	21	180	3780
3X	SB	SB	1	21	25	525	60	31500
SB	3X	SB	25	1	25	625	60	37500
SB	SB	3X	25	21	1	525	60	31500
3X	SB	DB	1	21	14	294	30	8820
SB	3X	DB	25	1	14	350	30	10500
SB	DB	3X	25	13	1	325	30	9750
3X	DB	SB	1	13	25	325	30	9750
DB	3X	SB	14	1	25	350	30	10500
DB	SB	3X	14	21	1	294	30	8820
3X	SB	TB	1	21	8	168	30	5040
SB	3X	TB	25	1	8	200	30	6000
SB	TB	3X	25	8	1	200	30	6000
3X	TB	SB	1	8	25	200	30	6000
TB	3X	SB	8	1	25	200	30	6000
TB	SB	3X	8	21	1	168	30	5040
3X	DB	TB	1	13	8	104	30	3120
DB	3X	TB	14	12	8	1344	30	40320
DB	TB	3X	14	8	1	112	30	3360
3X	TB	DB	1	8	14	112	30	3360
TB	3X	DB	8	1	14	112	30	3360
TB	DB	3X	8	13	1	104	30	3120
TB	TB	TB	8	8	8	512	80	40960
DB	DB	DB	14	13	14	2548	40	101920
SB	SB	SB	25	21	25	13125	20	262500

3X	3X	BLANK	1	1	73	73	20	1460
BLANK	3X	3X	74	1	1	74	20	1480
3X	BLANK	3X	1	77	1	77	20	1540
3X	BLANK	BLANK	1	77	73	5621	4	22484
BLANK	3X	BLANK	74	1	73	5402	4	21608
BLANK	BLANK	3X	74	77	1	5698	4	22792
3X	BLANK	SB	1	77	25	1925	4	7700
SB	3X	BLANK	25	1	73	1825	4	7300
BLANK	SB	3X	74	21	1	1554	4	6216
3X	SB	BLANK	1	21	73	1533	4	6132
BLANK	3X	SB	74	1	25	1850	4	7400
SB	BLANK	3X	25	77	1	1925	4	7700
3X	BLANK	DB	1	77	14	1078	4	4312
DB	3X	BLANK	14	1	73	1022	4	4088
BLANK	DB	3X	74	13	1	962	4	3848
3X	DB	BLANK	1	13	73	949	4	3796
BLANK	3X	DB	74	1	24	1776	4	7104
DB	BLANK	3X	14	77	1	1078	4	4312
3X	BLANK	TB	1	77	8	616	4	2464
TB	3X	BLANK	8	1	73	584	4	2336
BLANK	TB	3X	74	8	1	592	4	2368
3X	TB	BLANK	1	8	73	584	4	2336
BLANK	3X	TB	74	1	8	592	4	2368
3X	7	SB	1	8	25	200	4	800
3X	SB	7	1	21	7	147	4	588
3X	7	DB	1	8	14	112	4	448
3X	DB	7	1	13	7	91	4	364
3X	7	TB	1	8	8	64	4	256
3X	TB	7	1	8	7	56	4	224
7	3X	SB	6	1	25	150	4	600
SB	3X	7	25	1	7	175	4	700
7	3X	DB	6	1	14	84	4	336
DB	3X	7	14	1	7	98	4	392
7	3X	TB	6	1	8	48	4	192
TB	3X	7	8	1	7	56	4	224
SB	7	3X	25	8	1	200	4	800
7	SB	3X	6	21	1	126	4	504
DB	7	3X	14	8	1	112	4	448
7	DB	3X	6	13	1	78	4	312
7	TB	3X	6	8	1	48	4	192
TB	7	3X	12	8	1	96	4	384
SB	SB	DB	25	21	14	7350	10	73500
DB	SB	SB	14	21	25	7350	10	73500
SB	DB	SB	25	13	25	8125	10	81250
SB	SB	TB	25	21	8	4200	10	42000
TB	SB	SB	8	21	25	4200	10	42000
SB	TB	SB	25	8	25	5000	10	50000
DB	DB	TB	14	13	8	1456	10	14560

TB	DB	DB	8	13	14	1456	10	14560
DB	TB	DB	14	8	14	1568	10	15680
DB	DB	SB	14	13	25	4550	10	45500
SB	DB	DB	25	13	14	4550	10	45500
DB	SB	DB	14	21	14	4116	10	41160
TB	TB	DB	8	8	23	1472	10	14720
DB	TB	TB	14	8	8	896	10	8960
TB	DB	TB	8	13	8	832	10	8320
TB	TB	SB	8	8	25	1600	10	16000
SB	TB	TB	25	18	8	3600	10	36000
TB	SB	TB	8	21	8	1344	10	13440
SB	DB	TB	25	13	8	2600	10	26000
DB	SB	TB	14	21	8	2352	10	23520
DB	TB	SB	14	8	25	2800	10	28000
SB	TB	DB	8	8	14	896	10	8960
TB	SB	DB	8	21	14	2352	10	23520
TB	DB	SB	8	13	25	2600	10	26000
TOTAL			1692	1579	1719	140811	16072	1941858

Pay Out Percentage:

With 128 virtual stop positions on each reel, there will be a total of
 $128 \times 128 \times 128 = 2,097,152$ cycles. The pay out percentage is calculated as
follow:

Cycle: 2,097,152

Pays = 1,941,858

Pay Out Percentage = (pays/cycle) x 100

$$= (1,941,858/2,097,152) \times 100$$

$$= 92.60 \%$$

The **Hold Percentage** by the casino is equal to

$$100 - \text{Pay Out Percentage}$$

$$= 100 - 92.6$$

$$= 7.40 \%$$

The hit frequency is calculated to be

Number of hits: 140,811

$$\textbf{Hit Frequency} = (\text{Number of Hits} / \text{Number of Cycle}) \times 100$$

$$= (140,811 / 2,097,152) \times 100$$

$$= 6.71 \%$$

$$\textbf{Win Frequency} = 1 / (\text{Hit Frequency})$$

$$= (1 / 6.71) \times 100$$

$$= 14.89$$

As we can see from this exercise, the casino will make a profit. We have taken an hypothetical slot machine and determined its percentage pay out, based on the number of virtual stops, the combinations of symbols aligning on a single line and the pay out for each winning combination. In the last section of this chapter, we will discuss random number generators and how these numbers are used to determine a winning combination .

Random Number Generator

At the beginning of this chapter, we mentioned that the random number generator is the most important part of the slot machine because it determines the outcome of every game. All modern slot machines are controlled by a computer, and one of the computer functions is to generate random numbers. However, computers are not random in any way at all, they follow a fixed sequence of instructions which is predictable. Starting from a set of conditions a computer will generate exactly the same sequence of random numbers time after time. In reality, there are very few things that

are truly random. Real random numbers are non-deterministic. A Geiger counter generates a pulse every time it detects a radioactive decay. The time between decays has a strong pure random component. The known exponential distribution can be easily removed mathematically but the mean time between decays in background radiation rises early in the morning and falls at night. There are non random patterns lying hidden in the data which could be exploited by an analyst attempting to reduce his uncertainty for the next number in the sequence. An electronic slot machine needs a sequence that is unpredictable to the left; that is, the player cannot determine the next spin based on pattern analysis of previous spins. Unpredictability to the right is not required since it makes no difference if the player could infer an earlier sequence from a later sequence. Pure random numbers are found in nature such as "quantum" events that happen inside individual atoms. For example, the gas inside a laser which stores bundles of "quanta" of energy and gives back as light. The time it takes for each atom to give back the energy is a random quantum event. Signal noises are also pure random events. In a slot machine, the random number generator is a mathematical formula that produces a long sequence of numbers that appear as random. Theoretically, if one knew the formula and what the last number was, one could calculate the next one and so on. To avoid this possibility, the slot machine runs its pseudo random number generator continually whether it is played or not, so nobody has any idea what number it has generated at any one time. The sequence for these numbers has a finite time and always repeat itself. A 32 binary number (2 raised to the 32 power) produces 4.3 billion numbers. If the slot machine runs the RNG 1000 times a second, a typical value, it will take 50 days to complete the sequence and start over. There are three types of random number generation methods:

1. Toy generators provided by most programming languages and many software packages. These RNG are used for small computer simulations such as waiting times for a queuing network.
2. Serious generators which use 32 and 64 bits are found in video games and in slot machines.
3. Truly random number generators which employ electronic circuits for measurements of natural phenomenon. These are used in the field of research (physics and even finance) and cryptography where security is vital for national defense.

For our purpose, we will examine the serious random number generators. Earlier, we said that most slot machines have 22 stop positions on each reel,

11 symbols and 11 spaces (blank). However, we saw that internally, they have hidden virtual stops with many more positions. These positions are list of numbers in the program of the computer. Each position is mapped to a stop on the physical reel, and when a particular number is chosen, the machine makes the physical reel stop at the corresponding place. When a coin or coins are inserted into the machine, the reels begin to spin and the computer pulls three numbers out of the subset of pseudo random numbers. They are very large numbers, anywhere between 1 and 4.3 billion. These numbers are then divided by the number of virtual stops and the remainder is kept in a memory block. Using our slot machine with 128 virtual stops, an "index" from 0 to 127 is used to look up a number in each virtual reel data and map it on the physical reel. The reduction from 4.3 billion to 127 means that every position on the virtual reel is pointed to by 34 million random numbers. Dropping a coin or hitting the button happens at a random time and the chances are very small that one can hit a particular number even if you knew it was about to come because the computer is generating numbers at a rate of about 1000 or more cycles per second.

The selection of numbers is taken from a computer program and using our previous example, we have set up table pay out based on those selected random numbers.

R1	R2	R3	Subset From	To	Weight	Probability	PAY
BLANK	BLANK	BLANK	0	6341725	6341726	0.0236248	0
BLANK	BLANK	SB	6341726	12683451	6341726	0.0236248	0
BLANK	BLANK	DB	12683452	19025177	6341726	0.0236248	0
BLANK	BLANK	TB	19025178	25366903	6341726	0.0236248	0
BLANK	SB	7	25366904	31708629	6341726	0.0236248	0
BLANK	DB	DB	31708630	38050355	6341726	0.0236248	0
BLANK	DB	TB	38050356	44392081	6341726	0.0236248	0
BLANK	TB	SB	44392082	50733807	6341726	0.0236248	0
BLANK	TB	DB	50733808	57075533	6341726	0.0236248	0
BLANK	SB	TB	57075534	63417259	6341726	0.0236248	0
BLANK	TB	TB	63417260	69758985	6341726	0.0236248	0
BLANK	TB	7	69758986	76100711	6341726	0.0236248	0
BLANK	DB	BLANK	76100712	82442437	6341726	0.0236248	0
BLANK	TB	BLANK	82442438	88784163	6341726	0.0236248	0
BLANK	SB	BLANK	88784164	95125889	6341726	0.0236248	0
BLANK	SB	SB	95125890	101467615	6341726	0.0236248	0
3X	3X	3X	101467616	101467747	132	0.0000005	2500
7	7	7	101467748	101571876	104129	0.0003879	200
3X	3X	7	101571877	101583275	11399	0.0000425	1800
7	3X	3X	101583276	101594674	11399	0.0000425	1800
3X	7	3X	101594675	101606073	11399	0.0000425	1800
3X	7	7	101606074	101640730	34657	0.0001291	600
7	3X	7	101640731	101675387	34657	0.0001291	600
7	7	3X	101675388	101710044	34657	0.0001291	600
3X	3X	TB	101710045	101738890	28846	0.0001075	720
TB	3X	3X	101738891	101767736	28846	0.0001075	720
3X	TB	3X	101767737	101796582	28846	0.0001075	720
3X	TB	TB	101796583	101893935	97353	0.0003627	240
TB	3X	TB	101893936	101991288	97353	0.0003627	240
TB	TB	3X	101991289	102088641	97353	0.0003627	240
3X	3X	DB	102088642	102153543	64902	0.0002418	360
DB	3X	3X	102153544	102218445	64902	0.0002418	360
DB	3X	3X	102218446	102283347	64902	0.0002418	360
3X	DB	DB	102283348	102456418	173071	0.0006447	120
DB	3X	DB	102456419	102629489	173071	0.0006447	120
DB	DB	3X	102629490	102802560	173071	0.0006447	120
3X	3X	SB	102802561	102917941	115381	0.0004298	180
SB	3X	3X	102917942	103033322	115381	0.0004298	180
3X	SB	3X	103033323	103148703	115381	0.0004298	180
3X	SB	SB	103148704	103494844	346141	0.0012895	60
SB	3X	SB	103494845	103840985	346141	0.0012895	60
SB	SB	3X	103840986	104187126	346141	0.0012895	60
3X	SB	DB	104187127	104884012	696886	0.0025961	30
SB	3X	DB	104884013	105580898	696886	0.0025961	30
SB	DB	3X	105580899	106277784	696886	0.0025961	30
3X	DB	SB	106277785	106974670	696886	0.0025961	30
DB	3X	SB	106974671	107671556	696886	0.0025961	30
DB	SB	3X	107671557	108368442	696886	0.0025961	30
3X	SB	TB	108368443	109065328	696886	0.0025961	30
SB	3X	TB	109065329	109762214	696886	0.0025961	30

SB	TB	3X	109762215	110459100	696886	0.0025961	30
3X	TB	SB	110459101	111155986	696886	0.0025961	30
TB	3X	SB	111155987	111852872	696886	0.0025961	30
TB	SB	3X	111852873	112549758	696886	0.0025961	30
3X	DB	TB	112549759	113246644	696886	0.0025961	30
DB	3X	TB	113246645	113943530	696886	0.0025961	30
DB	TB	3X	113943531	114640416	696886	0.0025961	30
3X	TB	DB	114640417	115337302	696886	0.0025961	30
TB	3X	DB	115337303	116034188	696886	0.0025961	30
TB	DB	3X	116034189	116731074	696886	0.0025961	30
TB	TB	TB	116731075	116990680	259606	0.0009671	80
DB	DB	DB	116990681	117309891	319211	0.0011892	40
SB	SB	SB	117309892	117899128	589237	0.0021951	20
3X	3X	BLANK	117899129	118488365	589237	0.0021951	20
BLANK	3X	3X	118488366	119077602	589237	0.0021951	20
3X	BLANK	3X	119077603	119666839	589237	0.0021951	20
3X	BLANK	BLANK	119666840	122798843	3132004	0.0116676	4
BLANK	3X	BLANK	122798844	125930847	3132004	0.0116676	4
BLANK	BLANK	3X	125930848	129062851	3132004	0.0116676	4
3X	BLANK	SB	129062852	132194855	3132004	0.0116676	4
SB	3X	BLANK	132194856	135326859	3132004	0.0116676	4
BLANK	SB	3X	135326860	138458863	3132004	0.0116676	4
3X	SB	BLANK	138458864	141590867	3132004	0.0116676	4
BLANK	3X	SB	141590868	144722871	3132004	0.0116676	4
SB	BLANK	3X	144722872	147854875	3132004	0.0116676	4
3X	BLANK	DB	147854876	150986879	3132004	0.0116676	4
DB	3X	BLANK	150986880	154118883	3132004	0.0116676	4
BLANK	DB	3X	154118884	157250887	3132004	0.0116676	4
3X	DB	BLANK	157250888	160382891	3132004	0.0116676	4
BLANK	3X	DB	160382892	163514895	3132004	0.0116676	4
DB	BLANK	3X	163514896	166646899	3132004	0.0116676	4
3X	BLANK	TB	166646900	169778903	3132004	0.0116676	4
TB	3X	BLANK	169778904	172910907	3132004	0.0116676	4
BLANK	TB	3X	172910908	176042911	3132004	0.0116676	4
3X	TB	BLANK	176042912	179174915	3132004	0.0116676	4
BLANK	3X	TB	179174916	182306919	3132004	0.0116676	4
3X	7	SB	182306920	185438923	3132004	0.0116676	4
3X	SB	7	185438924	188570927	3132004	0.0116676	4
3X	7	DB	188570928	191702931	3132004	0.0116676	4
3X	DB	7	191702932	194834935	3132004	0.0116676	4
3X	7	TB	194834936	197966939	3132004	0.0116676	4
3X	TB	7	197966940	201098943	3132004	0.0116676	4
7	3X	SB	201098944	204230947	3132004	0.0116676	4
SB	3X	7	204230948	207362951	3132004	0.0116676	4
7	3X	DB	207362952	210494955	3132004	0.0116676	4
DB	3X	7	210494956	213626959	3132004	0.0116676	4
7	3X	TB	213626960	216758963	3132004	0.0116676	4
TB	3X	7	216758964	219890967	3132004	0.0116676	4
SB	7	3X	219890968	223022971	3132004	0.0116676	4
7	SB	3X	223022972	226154975	3132004	0.0116676	4

DB	7	3X	226154976	229286979	3132004	0.0116676	4
7	DB	3X	229286980	232418983	3132004	0.0116676	4
7	TB	3X	232418984	235550987	3132004	0.0116676	4
TB	7	3X	235550988	238682991	3132004	0.0116676	4
SB	SB	DB	238682992	239922677	1239686	0.0046182	10
DB	SB	SB	239922678	241162363	1239686	0.0046182	10
SB	DB	SB	241162364	242402049	1239686	0.0046182	10
SB	SB	TB	242402050	243641735	1239686	0.0046182	10
TB	SB	SB	243641736	244881421	1239686	0.0046182	10
SB	TB	SB	244881422	246121107	1239686	0.0046182	10
DB	DB	TB	246121108	247360793	1239686	0.0046182	10
TB	DB	DB	247360794	248600479	1239686	0.0046182	10
DB	TB	DB	248600480	249840165	1239686	0.0046182	10
DB	DB	SB	249840166	251079851	1239686	0.0046182	10
SB	DB	DB	251079852	252319537	1239686	0.0046182	10
DB	SB	DB	252319538	253559223	1239686	0.0046182	10
TB	TB	DB	253559224	254798909	1239686	0.0046182	10
DB	TB	TB	254798910	256038595	1239686	0.0046182	10
TB	DB	TB	256038596	257278281	1239686	0.0046182	10
TB	TB	SB	257278282	258517967	1239686	0.0046182	10
SB	TB	TB	258517968	259757653	1239686	0.0046182	10
TB	SB	TB	259757654	260997339	1239686	0.0046182	10
SB	DB	TB	260997340	262237025	1239686	0.0046182	10
DB	SB	TB	262237026	263476711	1239686	0.0046182	10
DB	TB	SB	263476712	264716397	1239686	0.0046182	10
SB	TB	DB	264716398	265956083	1239686	0.0046182	10
TB	SB	DB	265956084	267195769	1239686	0.0046182	10
TB	DB	SB	267195770	268435455	1239686	0.0046182	10

TOTAL					268435456	1.0000000	16072
--------------	--	--	--	--	------------------	------------------	--------------

The fundamental function of a random number generator can be shown in FIG. 1. At any time instant when the start button is actuated, a random number is sampled. The sampled value is in turn referred to a winning probability table to generate a request. Then referring to the symbol arrangement table and the winning table, each reel R1 to R3 is controlled so as to provide a win corresponding to the request.

Internal Controls

Gaming Control Boards (GCB) have developed strict planning and rigorous internal control of casino audits. Because of limited documentation of casino revenues, analytical reviews are used to test casino revenues. This is accomplished by comparing slot machines theoretical hold results to the actual results. As we mentioned earlier, theoretical hold or win as computed by reference to its payout schedule and reel setting are used as standards for the casinos. Deviation of the actual hold from theoretical hold percentage is a sign of either mechanical electrical or other performance problems. Some of the minimum requirements include testing of the following:

- 1) Coin in meter is functioning correctly
- 2) Coin in meter is read at least weekly and review for correctness
- 3) Theoretical/actual hold comparison made once a month
- 4) Statistical report reviewed by slot managers monthly
- 5) Any variation are to be investigated and resolved in a timely manner

These internal control are established by casinos in order to comply with GCB inspectors and if not followed, casinos may loose their license. Our next chapter will take us inside the slot machine and provide the reader with a detail description of the many components that make up a modern three reel slot machine. Our next chapter will take us inside the slot machine and provide the reader with a detail description of the many components that make up a modern electronic slot machine.

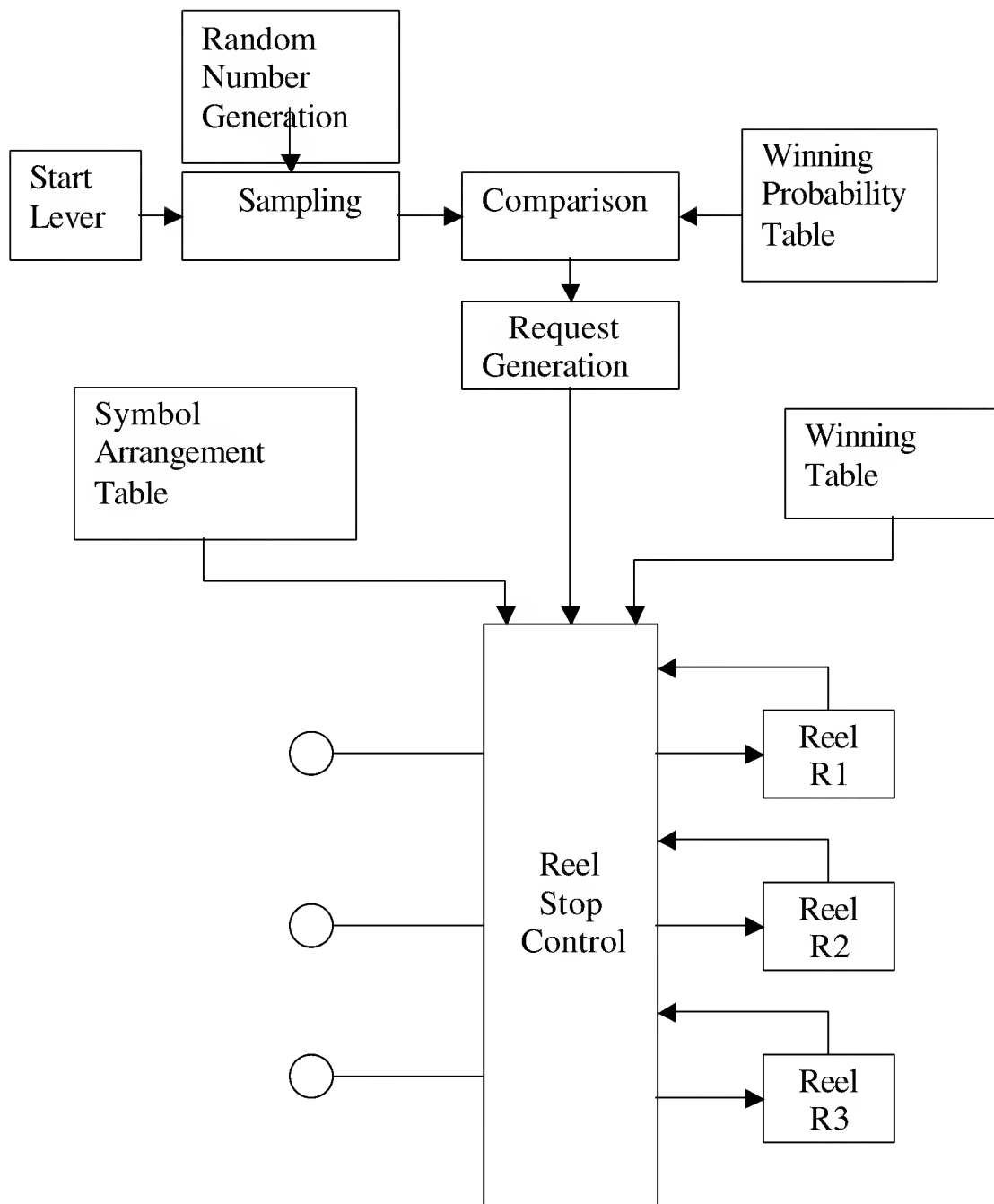


FIG. 1 Block diagram showing the functions of a random generator in a slot Machine

CHAPTER III

The Electronic Three Reel Single Line Slot Machine

Standard Model

A slot machine is essentially an electro-mechanical device. FIG 2. shows the basic external components of a slot machine. First we have the slot machine cabinet which comes in various styles and sizes, from tall upright machines to low narrow body machines. The top part of the cabinet is named the top box . This is where the Pay Table Glass (FIG. 3) is located and winning combinations are displayed. A bright light illuminates the Pay Table. Below the top box and covering the front of the machine we have the slot door which protects the inside of the machine. The lock is well secure so it will not easily be opened. There are two other illuminated display glasses, the reel glass and the called belly glass (FIGS. 4 & 5). The reel glass covers the reels of the machine, indicating a single line or multiple lines where the reel symbols have to line up to make a winning combination. The belly glass is a decorative display glass which advertises the type of game being played. Located between the reel glass and the belly glass is a button panel where the buttons are situated as well as the coin insert mouth. Most machines have a bet one button, a max button, a spin button and a coin attendant button. The size of the coin insert mouth varies according to the size of the coins played. A bill acceptor is situated next to or above the belly glass, enabling one to insert bills instead of coins. Also, a play card insert is located for the player to keep a running count of the number of cycles played on that machine. The coin tray is located below the belly glass. To the right of the slot machine, we find the handle which operates the same spin button found between the reel glass and the belly glass. The handle is there for cosmetic reason only and does not influence the outcome of the game in any way. On top of the cabinet, we have the tower light. This light has two shades of color. One is for the jackpot and the other is for malfunctions which requires the assistance of a floor attendant. The colors also describes the type of coin denomination being played.

Operation

To understand the basic operation of modern slot machine we will be using block diagrams throughout this section. A picture is worth a thousands words and hopefully this method will greatly simplify our task without going to detail technical descriptions. Upon opening the door of a slot machine, we find a neat arrangement of electronic and mechanical components. Modern three reel-type slot machines evolved from mechanical type machines wherein mechanical clutches were relied on to stop spinning display reels at random locations to display a game result to today's machine where a microprocessor randomly selects a game result. If we remember that by generating a greater number of random numbers than the number of available sets of different symbol combinations, and by varying the sizes of the subsets of random numbers which correspond to each different set of game symbols then the probability of a particular game result can be predetermined for the machine. The random number generator is the brain of the slot machine.

The basic operation of a slot machine is functionally illustrated in FIG. 6. Coins are inserted into the slot (1) and detected by the coin acceptor (2). The coins pass along a conduit into a coin payout hopper (3). The level of coins in the payout hopper is sensed by a detector which when full causes the coin diverter mechanism (4) to deflect a paddle and divert the coins recorded into a coin counter to come to rest in a coin drop box (5). When the coins have been accepted, a signal energizes the game to start. If multiple coins are fed into the machine a circuit detects this fact and signals the control and payout logic of the number of coins inserted. Once the play button is pressed or the handle is pulled (6), a shaft is rotated to power the reel mechanism (7). A random number generator (8) programs selected position, causes the individual reel to stop and determines a payout. When the payout is complete, a signal passes to the main control unit and a new game is activated. The control operation is illustrated in FIG. 7.

The hopper (3) is responsible for the paying out of coins into the coin tray. A hopper is a strong motor that turns a metal wheel. The wheel is a solid circular plate with raised pins that are spaced evenly apart towards the outer edge of the wheel. The distance between the pins is determined by the size of the coins that the hopper is required to pay out. The inner section of the wheel is also raised to enable a coin to rest on the wheel without slipping off. This wheel is large enough to hold up to a thousand coins. All the coins are controlled by the MPU, that is, how many coins are needed to be paid out. An optical sensor counts the coins as they leave the hopper. When the

last coin has been counted, the computer sends a signal to the hopper motor brake to engage and instantly forces the motor to stop. The machine accepts the coins and verifies that they correct. This is done using a coin comparator. The coin is compared with a sample coin. Using magnetic coils, an electronic measurement can be determined to assess the magnetic properties of the inserted coins to that of the sample coin. Usually, there will be three coils placed strategically near the coin insert mouth. The sample coin will register a magnetic signal and when a coin is inserted, its property will be compared and information send to an electronic comparator. If the two signals are identified, then the coin will be allowed to pass through and start the game, otherwise a negative output will activate a solenoid to block the path of the coin and send it to the coin tray. Once, the coin is accepted, it will travel down and pass through two optical sensors. One sensor measures direction and the other time of travel. Should the direction change or the time vary from a predetermined time, the machine will activate an alarm and require the assistance of an attendant to rectify the situation.

FIG. 8 illustrates the process in selecting a random number. The random number generator selects a random number within a predetermined range of random numbers. For example, in a 32 bit system the number may be one of 4,294,967,396 numbers in a range of 0 to 4,294,967,396. Then, this number is utilized in conjunction with a stored look-up table in an memory device called an EPROM (Erasable Programmable Read Only Memory) to select a game result comprising the three reel symbols S1,S2,S3. Next all three reels are made to spin. The first reel continues to spin for a first predetermined free spin period, typically one second, and upon completion a stopping procedure is initiated whereby signals developed by the game selected virtual stop are compared with signals corresponding to the desired game symbol S1. The stepper motor is interrupted and a stop routine is initiated to stop the display reel with the desired symbol displayed. In the mean time the second display reel continues to spin, and upon completion of a second predetermined spin period, also around one second, the signals generated by the symbol on reel 2 are compared with signals corresponding to the desired game symbol S2 and upon acceptance, a stop routine is initiated causing reel 2 to stop. Similarly, the third reel continues to spin for about a second and again a predetermined symbol initiates the stop. In the event of a spin error, the position of any one of the three reels, either as a result of the stepper motor slipping or failing to stop in response to a stepper pulse, an alarm is sounded and the game is stopped. If there are no spin errors, a determination is made whether the game results constitute a win and if so the

hopper mechanism is actuated to provide a pay out.

Thus far, we have gone through the operation of a slot machine and have a better understanding of how all the components fit together. The question a reader may have at this point of this book is how can we use this knowledge to increase our odds at playing slots. Well, there is a way to theoretically increase our odds in winning a jackpot. First, we need to have a detail record of a given machine. Specifically, a knowledge of the payout that has gone for the life cycle of this machine, since theoretically a given hold percentage will eventually bottom out and a winning number will appear. This period of time will depend on how long the machine has been active and the cycle of the machine. A machine with a low cycle will take a shorter time to complete a cycle than one with a larger cycle. For instance a slot machine with a 32 stop per reel being played six times a minute, 24 hours a day will take

$$(32 \times 32 \times 32) / \{ 6 \text{ play/min} \times 60 \text{ min/hr} \times 24/\text{day} \} = 3.79 \text{ days}$$

to complete a full cycle, while a machine with 128 stops per reel will take

$$(128 \times 128 \times 128) / \{ 6 \text{ play/min} \times 60 \text{ min/hr} \times 24 \text{ hr/day} \} = 242.7 \text{ days}$$

and for a machine with 256 stops per reel will take 5.32 years to complete its cycle of 16,777,216 combinations. Unfortunately, this information is available only to casino owners.

The Law of Averages

One of the most important results in probability theory is the law of averages, also known as the law of large numbers. This mathematical law provides an important connection between theoretical probabilities and observed rules of random events. The law is summarized as follow:

In repeated, independent trials of the same experiment, the observed fraction of occurrences of an event eventually approaches its theoretical probability

Basically it says that if you repeatedly play an "unfavorable" game, even though you are uncertain of the results of an individual play, in the long run you will surely be a loser. How can we then make slot playing a profitable game. Well, the answer is found in common sense and here are simple rules that one can follow and reduce the odds of being a loser. The following guidelines will be helpful:

- 1) Play a high denomination machine if you can afford it for the higher the denomination the higher the payout percentage.*
- 2) Keep your winnings from your buy-in amount, i.e. any money that you win keep aside with the only money you bet with being the money you budgeted for.*
- 3) Play machine with a lower cycle. There is no way of knowing which machine has a short cycle but the machines with lower jackpot amounts generally have lower cycles.*
- 4) Play machines that have "wild" symbols such as Double Diamonds, Triple Diamonds, Five Times Pay. These are the most popular machines among players and can increase you pay off.*

Technological Advancements and Future Development

Computer technology is growing rapidly giving the slot machine industries new opportunities for creating better graphics, better sounds and easier to use interactive games. Multigame machine make switching between games and challenge levels as easy as the touch of a finger. With upgraded machines electronics, state of the art video game machine offer many choices to the players. From simulated spinning reels to video blackjack, players have numerous choices within a single machine. Another innovation to come out as a result of new technology is the bonus game. These are available on both spinning and video machines. The attraction lies in the overall game experience with exciting bonus opportunities for the players. Combining multimedia features, digital stereo sound, and interactive game play, machines like Wheel of Fortune, Elvis and the Adams Family are taking games to a new level.

The next wave of technology will feature a coin less system that pays out in vouchers instead of coins. This will be a great saving to casino owners. There will no longer be a need for employees to empty machines night after night, count and sort coins, and reduce the number of attendants who take care of hopper fills and coin jams. The only set back for the player is that the game will be played faster and your risk will increase.

Players will be attracted by these new electronic gaming and it will become more important than ever to learn how to become in control of the game. This will be presented in the next chapter.



FIG. 2 Electro-Mechanical Slot Machine

1 ST COIN WINNERS		2 ND COIN WINNERS	
  	1000	  	2,500
7 7 7	100	7 7 7	200
  	40	  	80
  	20	  	40
   /  	10	   /  	20
  	5	  	10
ANY ONE 	2	ANY ONE 	4

 WINNINGS ARE PAID IN CASH
 WINNINGS ARE PAID IN CASH
 WINNINGS ARE PAID IN CASH
 WINNINGS ARE PAID IN CASH

1. WINNINGS ARE PAID IN CASH
2. WINNINGS ARE PAID IN CASH
3. WINNINGS ARE PAID IN CASH
4. WINNINGS ARE PAID IN CASH

FIG. 3 Pay Table Glass



FIG. 4 Reel Glass and Button Panel



FIG. 5 Belly Glass

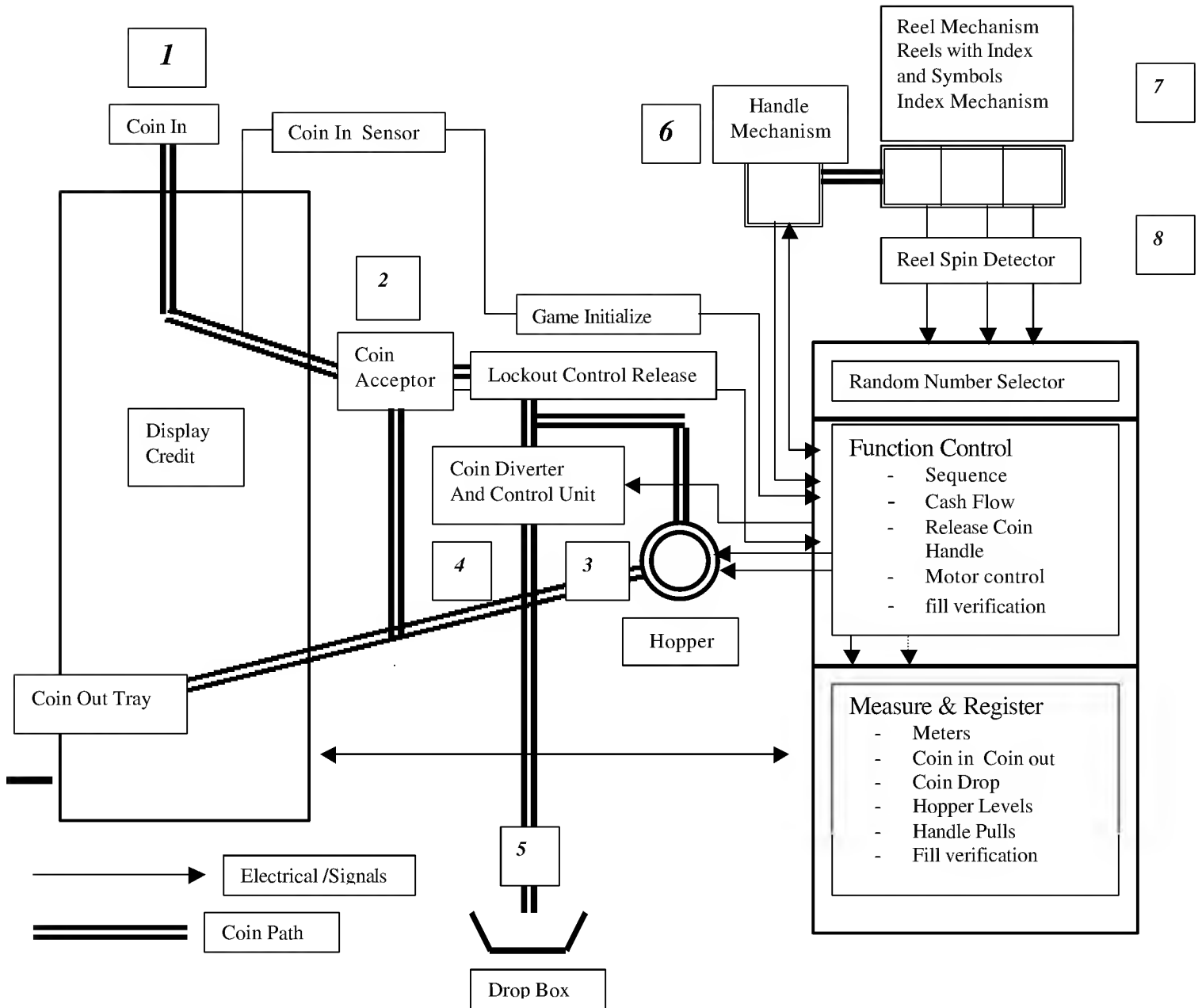


FIG. 6 Basic Operation of Electronic Slot Machine

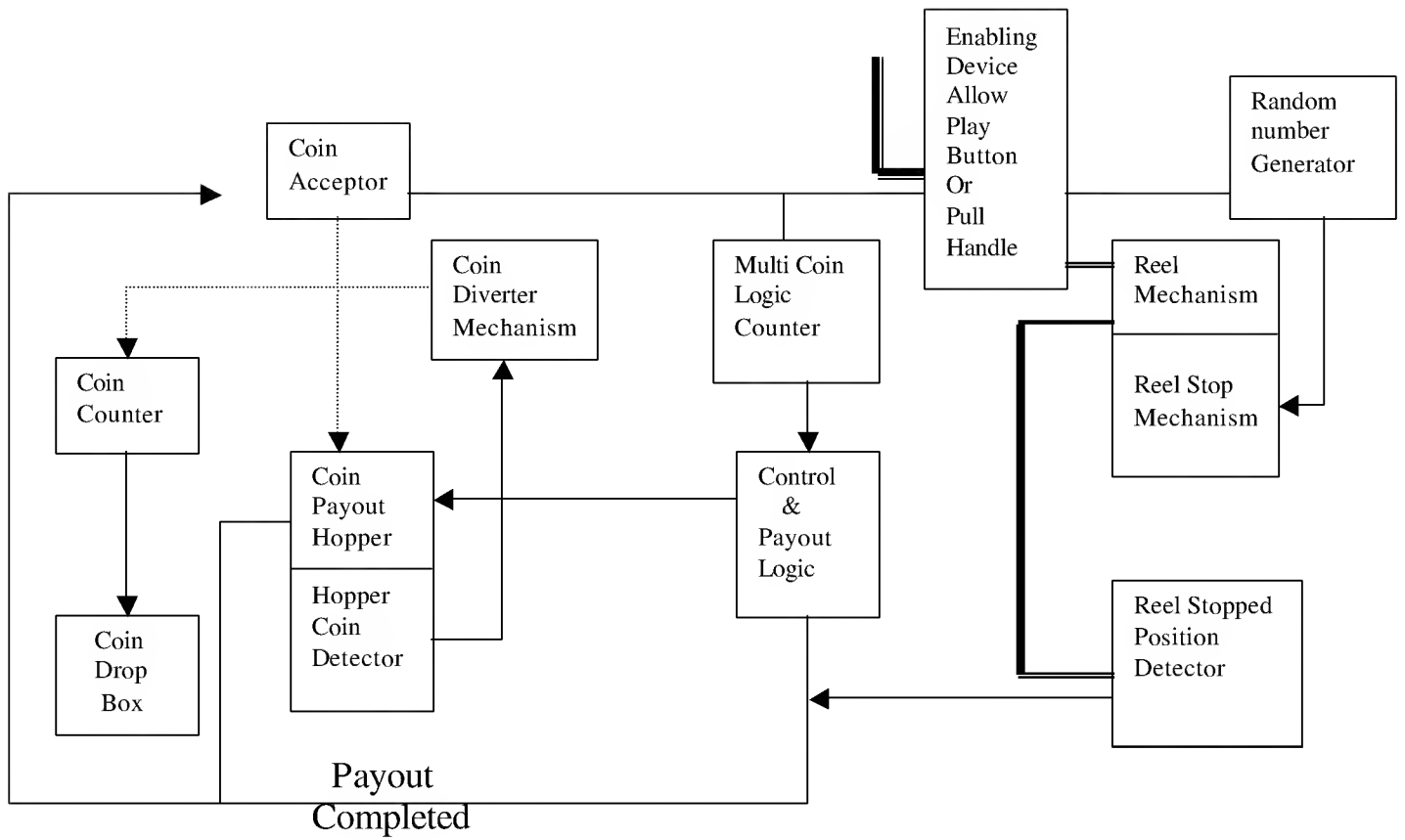


FIG. 7 Control system Diagram

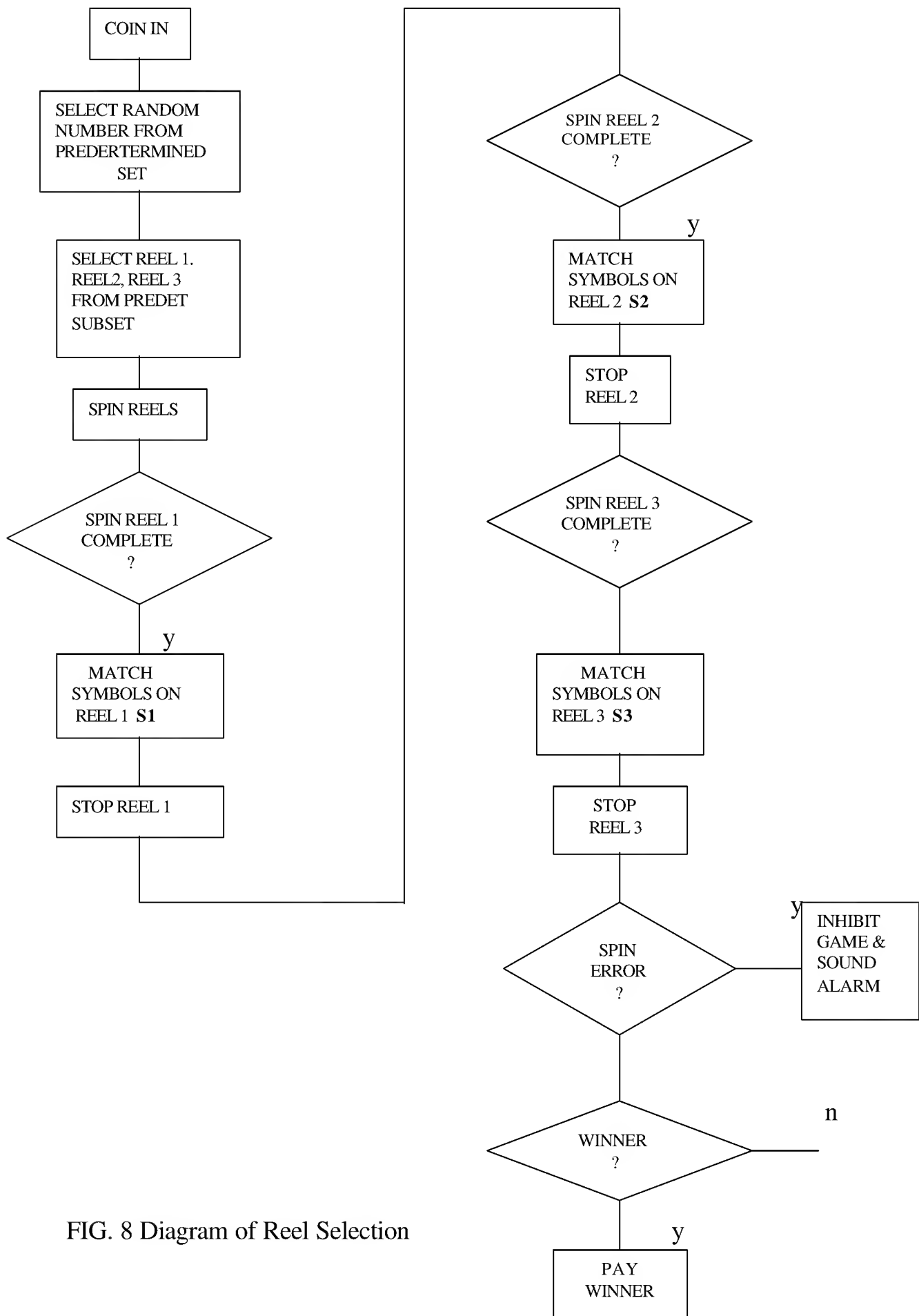


FIG. 8 Diagram of Reel Selection

CHAPTER IV

Slot Machines and Human Behavior

Conditioning and Reinforcement

As a result of technological advances and a variety of reasons, slot machines have become the most popular form of casino-style gambling in the United States and in many other countries over the past two decades. This is reflected in both revenues and total number of machines. In 1997, there were nearly a half million slot machines in the United States. (Salomon Smith Barney, 1998). The rise of popularity with slot machines may be found in the machines' potent control over the individual player. According to psychologists studying this problem, slot machine gambling demonstrate the principle of conditioning and superstitious behavior that contributes to the effectiveness of slot machines. The behavior generated by slot machines is so potent that it is often regarded as addictive. We will examine some of these controlling factors and learn how we can help our self to stay in control and be one step ahead of the game.

When we play a slot machine, we find that it generates a high rate of behavior for a long period of time. The machine's sole purpose is to be operated. Other machines either produce a product or help us to complete tasks more quickly, easily and accurately. In contrast, slot machines are simply there to be operated and meet their owner's objective. The first question we must ask is what makes this machine so successful?

The successful slot machine must be attractive, induce operation and maintain the player's behavior even though the player is operating at a loss. In the late nineteenth century, slot machines were works of art. The exquisite tool work, vivid colors and visible jackpot all draws the potential player to the machine. Today's technology offers stereo sound, super graphics and interactive choices which induces the operation of the slot machine and maintains the player's interest even further. The players behavior is induced by the pay out and the possibility of winning big. People travel to Las Vegas in search of hitting the jackpot. Once, the action is initiated, the player maintains this action by hoping that the next outcome will bring him fortune.

As we have seen in Chapter III, winning combinations of symbols are predetermined and in general small pay out maintain the action. Getting two out of three jackpot symbols on the pay line is exciting. The player has been reinforced at no cost to the machine owner. We must also mentioned the way in which reels stop enhances the reinforcement. If the first reel stops on a potential winner, a heightened state of arousal occurs. Symbols above and below the pay line also condition the mind of the player. Not only is the machine conditioning the mind, but the environment of the casino as well. People near us and around us are engaged in the same behavior and they all are interested in the same outcome, a jackpot. The screams of joy and excitement can be heard whenever jackpots are announced. All these produce stimuli that maintain the schedule of reinforcement.

Superstition is another great generator of behavior reinforcement. Players develop strange ways of pulling the handle, standing, putting money in the coin slot, rubbing the side of the machine, etc. From Chapter III, we know that no such behavior can influences the outcome of the game but in a player's mind these actions reinforce his or her conviction that the machine will hit. Another form of slot conditioning is playing more than one machine simultaneously. When this happens, not only slot playing is reinforced but so is switching. Soon, every response becomes a switching response and again the need to continue to play is reinforce.

Environmental Factors

Casino floors are carefully created to attract gamblers. Sounds of bustling players, casino employees, loud machines, ringing bells, shouts of joys, bright lights, all contribute indirectly in shaping the players' behavior and habits. So, if you are going to gamble with slot machines, take time to observe your surroundings and learn to read the pay schedule . As we pointed out in Chapter III, the higher the jackpot pay out, the higher the number of virtual stops on a reel and a lower probability of hits. Look for a machine that has a lower jackpot pay out, for example, a machine that has a jackpot of 20,000 coins for a three coin play will more likely have a lower probability of hits than one that has 2500 coins jackpot. Popular machines such as IGTs Double Diamond and Triple Diamonds is a good beginning for a novice slot player. Playing the maximum coins guarantees a higher return if you win but also depletes your bank roll at a faster rate. If your goal is to extend your playing time, then play coins one at a time followed by a perhaps a few and in between maximum coin play. Remember, the computer

is randomly selecting a 1000 numbers per second and there are millions of numbers to choose from and the occurrence of a big jackpot win is possible but unlikely for the time you spend near the machine. One must remember that the length of time it takes for a full cycle to complete may be as short as a week or as long as five years. Since a winning combination is solely determined from a selection of a random number, it is possible for a jackpot to appear back-to-back although this may be rare.

We initially said that the better machines, those that have a higher percentage pay out may be placed near the doors or restaurants to attract attention and bring more players, the reality is that these machines placement's are done first for appearances. All slots machines produce reports on a regular basis and machines that are not getting enough play will be moved to a different location, hoping to improve the number of plays. Therefore, placement is not based on a machine's percentage or payback percentage. It is based on how well a game is getting played, that is, the number of coins that are dropped into the machine. It is worth reminding the reader that changing any machine's configuration such as the pay out table is very time consuming and expensive to the casino owners and for this reason, the placement of machines is based strictly on the return of the machines rather than percentage.

Remarks

An electronic slot machine returns between 85 and 100 percent, depending on the type of game and coin denomination. The average house advantage is calculated to be around nine percent for all machines. Some may pay more and some less. The first lesson for anyone gambling with a slot machine is to read and become familiar with the glass pay table schedule and recognize the symbols that constitute a jackpot. Always play the maximum bet whenever you can because this gives you the best odds of winning long-term if you get a good hand and also with progressive jackpots the maximum bet is required to have any chance of collecting the jackpot if you are lucky enough to hit it.

Slot playing is very easy and no skills are required except for machine identification. For example, you may see two similar machines, one offers a progressive jackpot while the other pays a standard pay out schedule. The right choice that comes to mind is to select the progressive machine because it offers a higher jackpot but when you consider the probability of hitting the

jackpot and the number of virtual stops per reel, you may want to think again and play the standard slot machine.

As you soon find out when you enter a casino, machines with a 99 percent payback are advertised and can be spotted on the floor with ease. Be careful, when you play these machines. There are only a “selected” number of these machines and may be limited to only a single bank of machines. It is your responsibility to find them and all you need to do is just ask a slot employee to locate one for you.

Play responsibly, that is, play what you can afford. If dollar slots are too much for your budget, drop down to a quarter machine. It’s always a better value to play four quarters versus one dollar or five nickels instead of a quarter. Again, read all the posted material on a slot machine. It is your responsibility to fully understand all printed information concerning the number of coins to insert, lines needed to be lit and prizes or awards. If a machine is cold, that is, does not hit, move to another one even if it’s your favorite. The longer you stay on any slot machine, the more time the machine’s built in mathematical advantage has to work against you.

Remember to set a limit when you are playing slots. Discipline yourself and use only your bankroll and save your winnings. If you are ahead one dollar, you have won. Avoid using your credit cards. They will only sink you further in debts. It is always important to know what you are playing for so study the machine’s pay chart. Play the maximum coins for best return whenever possible.

In summary, the key to winning is quitting while you are ahead. Even though a slot machine seems to be paying off, the longer you play, the more money you will lose, because it is programmed to do this. Remember, if you can leave with more money than you started with, even if it’s a dollar, you have won.

CHAPTER V

Conclusion

Now that we have a good idea of how slot machines work, the question we must ask is " Can you win ? ". In my judgment, the answer is " YES ". Of all the hundreds of people playing inside a casino, there will be a few winners and many losers. Even though, the probability is small that you will hit a jackpot whenever you insert a coin in the machine, the thing to remember about slot machines is that the random number inside the machine, controls your outcome. If you happen to hit a winning combination, it is the random number that predetermined your win, and no matter how small your chances are, there is still a finite probability that it can happen to you. Again, it is the game of chance and taking risks that controls the situation.

To test this concept, the author took a trip to Las Vegas, NV. in the spring of 2001 and spend two days and nights at the MGM Grand Resort and Casino. The first night, while playing a five coins quarter IGT Double Diamond machine, I hit a jackpot worth 3200 coins or \$800.00. This luck was primarily due to playing this machine at the right time and place. This occurred about 11:00 PM near the main lobby of the MGM where traffic is high and the machine was at the end of an aisle. Again, the theory that so called "loose slots" are located near high traffic and at the end of the aisles has some validity to it, but, through observations, I noticed that "loose slots" are placed randomly and pay frequent but small payoffs. I also believe, that the random number selection process may have more to do with it than mere location. The cycle of pay for that machine was higher than usual and for this reason, I continued to play and the jackpot was hit. The next day, I played again at the MGM and won several small wins consisting of 40 to 100 dollars, all with quarters. Outside the hotel, the weather was beautiful, a clear blue sky and temperatures in the mid sixties so I decided to check the other casinos and try my luck. I walked to the New York, New York casino, just across the MGM, and played there for an hour. Unfortunately, after loosing a hundred fifty dollars, I decided to move on to other hotels and casinos. Each time, I moved from place to placer, my losses increased with each move. Eventually, I sought refuge back to the MGM and took a lunch break.

Lesson learned #1: ***If you are going to play slot machines, refrain from hopping casinos to casinos, looking for that big hit.*** All slot machines function on the same principle even though payoff probabilities differ from one machine to another, I recommend that you find your favorite casino and play there. It will save you a lot of walking and frustration. On the next day, I continued to play and just as the mathematics have predicted, the longer you play, the quicker you will deplete your earnings.

Lesson learned #2: ***Put away your winnings and play only with your initial bank roll.*** The best way to keep from spending your money is to know, at all times, where you stand. Limit the number of play per machine without a win. I have experienced slot machines spin 30 times before a win came up and usually it was a small payoff. On the other hand, if the machine is paying off more than usual, stay with it until it stops paying after ten spins. In the two days of playing slots feverishly, I found myself winning and losing. The temptations were high and disciplining yourself to stop playing was even harder.

Lesson learned #3: ***If you are on a losing streak, STOP, and take time out.*** This is the only way you can gain control of yourself and the 'One Armed Bandit'. Do not try to justify your playing to regain your losses. Remember, the longer you play, the more you will lose.

Lesson learned #4: ***Play the right slots.*** Whatever denomination you choose to play, stick to the 2 coins, 3 reel non-progressive, single-pay line slot with the lowest top jackpot (1000 to 5000 coins). The lower the top jackpot, the more frequency the smaller paybacks and the better the odds.

Lesson learned #5: ***Don't play with the jackpot in mind.*** To win the jackpot, you must always bet the maximum coins which will run you out of money very fast. Your goal should be to find a slot that gives you the longest playing time and smaller but more frequent paybacks. The longer you can make your bankroll and playing time last, the better chance you have of finding the right slot machine at the right time. Learn to be satisfied with any win you can get.

Lesson learned #6: ***Know which slot locations to try.*** Every casino wants players to see other players winning. It's good advertising and enhances the client to play even more often. The better-paying machines are often put in heavy traffic, highly visible locations with room for crowds to gather and

cheer winners on. Such locations include crosswalks, elevated carousels, and banks of slots near the casino bar, lounges, change booth and coffee shops. It works most of the time.

Lesson learned #7: **Luck.** There is no "right" or "wrong" way to bet on a slot machine. If like most people, you go to a casino to get away from every day stress, it doesn't matter how you bet: maximum, minimum, or a combination of both. Ignore any "expert" or casino know-it-all who say to always bet maximum coins. It's your money and you are in control of how that money is being wagered. Remember the law of large numbers. The longer you play the more likely you will lose your winning.

Finally, my experience in playing slots was gratifying and exciting. I started out with one thousand dollars and was able to pay off all my expenses and still keep two hundred dollars. All this playing, approximately twenty hours, accumulated 1700 points on the MGM Grand Directors Card. This was a lot of playing and a great experience that will last for a while. To summarize, playing slot machines is exciting and fun but the game is costly and should be done only when the means for playing are there and for most of us, this translates into a few casino trips a year.

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